Control Strategies for Complex Movements Derived from Physical Systems Theory

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1. Introduction

The work presented here is part of an effort to develop a unified approach to the control of a system which may interact dynamically with its environment. The principal perceived problem is that when a controlled system interacts with its environment, its performance may be drastically altered. Even if the controlled system is stable in isolation, when it interacts with dynamic objects in its environment that stability may be jeopardised. This problem is particularly actue for a manipulator. Manipulation has been succintly described as a series of collisions between the manipulator and the objects in its environment [7]. Every time a manipulator grasps or releases an object, the dynamic behaviour of the physical system interfaced to the controller undergoes an abrupt change, and this change may have a profound effect on the manipulator's behaviour.

The approach discussed here is based on physical systems theory, and has been developed from an investigation of the strategies used to control the primate upper extremities, [2, 11, 15, 16, 21] and the application of similar strategies to the control of robot manipulators [12, 13, 14, 17, 18]. It may be sufficiently general to have application for controlling other complex biological systems. The ultimate goal of this work is to develop a class of controlled systems which could be dynamically coupled to or isolated from a wide variety of environments without serious degradation of performance and stability. This paper will show that the preservation of stability in the face of changing environmental dynamics can be achieved through a control strategy which ensures that a manipulator's behaviour is compatible with the physical behaviour of its environment.

2. Physical Equivalence

The basis of the approach is the concept of physical equivalence [12]. Any controlled system will consist of "hardware" components (e.g. sensors, actuators and structures) combined with controlling "software" (e.g. a neural network, brain or computer). A unified approach to the analysis and design of both the controller and the physical hardware can be developed by postulating that, taken together, the hardware and software is still a physical system in the same sense that the hardware alone is.

The value of this conjecture is its implication that no controller need be considered unless it results in a behaviour of the controlled system, which can be described as an equivalent physical system. Several well developed formalisms exist for describing physical systems, the most notable being Paynter's bond graphs [24, 27], which have been applied successfully to a broader class of systems than any other formalism. The postulate of physical equivalence justifies using the same technique to describe control systems. This provides a powerful and intuitive way of thinking about control action in physical terms, and may provide an effective vehicle for promoting communication between control system theorists and those working in other disciplines.

However, if this conjecture is to be of anything more than philosophical interest, it is necessary to clarify the definition of a physical system. What (if anything) distinguishes the differential equations used to model

H. Haken (ed.), *Complex Systems — Operational Approaches in Neurobiology, Physics, and Computers*

a physical system from any other general system of differential equations? No complete definition is attempted here, but some key issues are considered. One of the important differences lies in the structure of the equations.

3. Structure

What is meant by structure and why does it matter? Consider the differential equations for a general second-order linear system driven by a single input.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u \tag{1}$$

x_1, x_2: state variables

u: input variable

a_{11}, a_{12}, a_{21}, a_{22}, b_1, b_2: system parameters

or

$$\dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x} + \mathbf{B}u \tag{2}$$

One important property of a system is its controllability, and this system is controllable if and only if the matrix $[\mathbf{B}|\mathbf{AB}]$ is of full rank.

One way of imposing structure on these equations is by restricting the values of some system parameters, and this can have a profound effect on system properties. Suppose, for example, that the parameters a_{12} and b_1 are identical to zero.

$$a_{12} \equiv 0 \tag{3}$$

$$b_1 \equiv 0 \tag{4}$$

The resulting system is structurally uncontrollable; it is always uncontrollable for all values of the remaining system parameters.

$$det[\mathbf{B}|\mathbf{AB}] = det\begin{bmatrix} 0 & 0 \\ b_2 & a_{22}b_2 \end{bmatrix} = 0 \tag{5}$$

If the differential equations are a mathematical model of a physical system, then that system will determine their structure. For example, dynamic interaction between a spring and a mass subject to external forces can be modelled by a second-order linear system of state equations. In this case the state variables can be given a physical meaning, and the equations may be written in phase-variable form in which one state variable is the displacement of the mass and the other its velocity.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k/m & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} F \tag{6}$$

x_1: displacement of mass

x_2: velocity of mass

k: spring constant

m: mass

F: external force

This system is structurally controllable. Aside from the trivial case $1/m = 0$ (corresponding to infinite mass) this system is always controllable for all values of its parameters.

$$det[\mathbf{B}|\mathbf{AB}] = det\begin{bmatrix} 0 & 1/m \\ 1/m & 0 \end{bmatrix} = -1/m^2 \tag{7}$$

This example shows that the structure imposed on the equations by the physical system they describe leads to useful restrictions on the behaviour they may exhibit.

4. Interaction

Another important characteristic of physical systems is the way they may interact. Consider two general first-order open linear systems. Each system receives an input from and delivers an output to its environment.

$$\dot{x}_1 = a_1 x_1 + b_1 u_1 \tag{8}$$

$$y_1 = c_1 x_1 \tag{9}$$

$$\dot{x}_2 = a_2 x_2 + b_2 u_2 \tag{10}$$

$$y_2 = c_2 x_2 \tag{11}$$

u_1, u_2: input variables
y_1, y_2: output variables
c_1, c_2: system parameters

The stability of each system in isolation is determined by the eigenvalues of its system matrix, in this simple case a scalar. A necessary and sufficient condition for assymptotic stability of each system is that its eigenvalue(s) be less than zero.

$$a_1 < 0 \tag{12}$$

$$a_2 < 0 \tag{13}$$

When the two systems are coupled the output of one becomes the input to the other.

$$u_1 = y_2 \tag{14}$$

$$u_2 = y_1 \tag{15}$$

The equations for the complete system are obtained by substitution.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} a_1 & b_1 c_2 \\ b_2 c_1 & a_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{16}$$

A condition for stability of the coupled system is:

$$a_1 a_2 - b_2 c_1 b_1 c_2 > 0 \tag{17}$$

Although the product $a_1\ a_2$ is greater than zero if the two systems are stable in isolation, stability of the coupled system requires that $a_1 a_2$ be greater than $b_2 c_1 b_1 c_2$. In general, stability of individual systems in isolation provides no guarantee of the stability of the system formed when they are dynamically coupled.

However, if the equations represent physical systems, then useful restrictions can be placed on the form of the coupling. In the formalism of bond graphs, dynamic interactions between physical systems are described (essentially by generalising Kirchoff's current and voltage laws) as an instantaneous exchange of energy without loss or storage [24]. Instantaneous energetic interaction or power flow between a physical system and its environment may always be described as a product of two variables, an effort (generalised voltage or force) and a flow (generalised current or velocity).

Energetic interaction between two systems also imposes a causal constraint on the forms of their input/output relations. One system must be an impedance, accepting flow (e.g. motion) input and producing effort (e.g. force) output while the other must be an admittance, accepting effort (e.g. force) input and producing flow (e.g. motion) output.

A mechanical spring and a frictional element experiencing a common force (i.e. in series) provides an example of an impedance; a mass and a frictional element sharing a common velocity provides an example of an admittance.

$$\dot{x}_1 = -k/b_1\, x_1 + V_1 \tag{18}$$

$$F_1 = k\, x_1 \tag{19}$$

x_1: spring displacement
V_1: input velocity
F_1: output force
k: spring constant
b_1: viscous friction constant

$$\dot{x}_2 = -b_2/m\, x_2 + 1/m\, F_2 \tag{20}$$

$$V_2 = x_2 \tag{21}$$

x_2: velocity of mass
F_2: input force
V_2: output velocity
m: mass
b_2: viscous friction constant

Assuming the usual convention [24, 27] that power is positive into a dynamic element or system imposes a sign constraint on the coupling equations. For example, if the coupling imposes a common velocity (flow) on the two systems, then to satisfy conservation of energy, the forces (efforts) must be equal but opposite (Newton's third law).

$$V_1 = V_2 \tag{22}$$

$$F_1 = -F_2 \tag{23}$$

The equations for the coupled system are again obtained by substitution:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -k/b_1 & 1 \\ -k/m & -b_2/m \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{24}$$

A condition for stability of the coupled system is:

$$(k/b_1)(b_2/m) + k/m > 0 \tag{25}$$

In this case, if the individual systems are stable in isolation, the coupled system is also stable. Physically, this makes sense as the stability of each system in isolation guarantees that its energy is always decreasing. Coupling the two systems does not generate any energy, therefore the total energy of the coupled system is also decreasing and the coupled system is stable. Again, a knowledge of the structure of the equations for a physical system permits stronger statements about its behaviour.

5. Impedance Control

The same concepts may usefully be applied to more complex systems. If a manipulator (biological or artificial) is to interact dynamically with its environment then it is important to understand the structure of the environmental dynamics and to ensure that the behaviour of the manipulator is compatible. In the vast majority of cases, the environment a manipulator grasps consists of inertial objects, possibly kinematically constrained, and may include some elastic and frictional elements. An environment of this class can be described using Lagrange's equations in the following form.

$$L(\underline{q},\dot{\underline{q}}) = E_k^*(\underline{q},\dot{\underline{q}}) - E_p(\underline{q}) \tag{26}$$

$$\frac{d}{dt}[\partial L/\partial \dot{\underline{q}}] - \partial L/\partial \underline{q} = -\underline{P}(\underline{q},\dot{\underline{q}}) + \underline{P}(t) \tag{27}$$

$\underline{q}$: vector of generalised coordinates
$L(\underline{q},\dot{\underline{q}})$: Lagrangian
$E_k^*(\underline{q},\dot{\underline{q}})$: kinetic co-energy
$E_p(\underline{q})$: potential energy
$\underline{P}(\underline{q},\dot{\underline{q}})$: generalised frictional forces
$\underline{P}(t)$: generalised input forces

The coupling between manipulator and environment is typically such that a set of points on the manipulator have the same position and velocity as a corresponding set of points on the environmental object. These points define an interaction port. The position and velocity of the interaction port of the environment are functions of its generalised coordinates.

$$\underline{X} = \underline{L}(\underline{q}) \tag{28}$$

$$\underline{V} = \mathbf{J}(\underline{q})\dot{\underline{q}} \tag{29}$$

$\underline{X}$: interaction port coordinates
$\underline{L}(\underline{q})$: kinematic transformation equations
$\underline{V}$: interaction port velocities
$\mathbf{J}(\underline{q})$: Jacobian of kinematic transformation

As the transformation from generalised coordinates to interaction port coordinates is non-energic, the generalised input force is related to the interaction port force through the transposed Jacobian.

$$\underline{P} = \mathbf{J}(\underline{q})^t\underline{F} \tag{30}$$

$\underline{F}$: interaction port forces

Thus the input/output relation at the interaction port is:

State equations:

$$\frac{d}{dt}[\partial L/\partial \dot{\underline{q}}] - \partial L/\partial \underline{q} = -\underline{P}(\underline{q},\dot{\underline{q}}) + \mathbf{J}(\underline{q})^t\underline{F} \tag{31}$$

Output equations:

$$\underline{V} = \mathbf{J}(\underline{q})\dot{\underline{q}} \tag{32}$$

These equations show that this class of environments accepts input forces and produces output motions in response. Note that in these equations the vector of generalised coordinates may be of any order and the Jacobian need not be square. It is not, in general, possible to reformulate the equations in the dual form with velocity as the input and force as the output; this system is a generalised mechanical admittance.

Accordingly, to be compatible with this class of environments, the manipulator should be a generalised impedance, accepting motion inputs and producing force outputs in response. As the behaviour of the manipulator or the demands of the task vary, that impedance may need to be modulated or controlled, and the approach outlined in this paper and elsewhere [12, 13, 14, 16, 17, 18] has therefore been termed impedance control. The principal distinguishing feature of this approach is in the objective of the controller. Conventional controllers are usually structured so as to make some selected time function of the system state variables (e.g. position, velocity, force, etc.) converge to a desired time function. For example, almost all of present robot control technology is focused on the problem of making the robot end effector follow a desired trajectory in space [23]. An impedance controller attempts the more demanding task of making the entire dynamic behaviour of the manipulator converge to some desired dynamic function relating input motions to output forces.

The feasibility of imposing a desired impedance on a robot manipulator has been demonstrated and discussed in detail elsewhere [13, 17]. It has been shown that if a robot controller is designed with an impedance as the target behaviour some of the more prominent computational problems associated with robot control – inversion of the robot kinematic equations and computation of the inverse Jacobian in the vicinity of singular points – can be eliminated. However, it is not the intent of this paper to discuss computational techniques,as their relevance to the general problem of control of complex systems is unclear. For example, in a biological system, computational complexity may not be a major issue.

Instead a more fundamental question will be addressed: Is it useful for a manipulator to assume the behaviour of a generalised mechanical impedance? As detailed elsewhere [12, 13, 14, 17] impedance control provides a unified framework for coordinating free motions, obstacle avoidance, kinematically constrained motions, and motions involving dynamic interaction. In this paper a further benefit of impedance control is considered: the preservation of stability in the face of changes in the dynamic environment to which a manipulator is coupled. One simple (but versatile) class of impedances produces an output force as a function of only the position and velocity of the interaction port. In the following it will be shown that if the manipulator has the behaviour of this general class of impedances then a sufficient condition for the manipulator and the environment to be stable in isolation from one another is also sufficient to guarantee that the coupled system formed by dynamic interaction between manipulator and environment is also stable.

6. Preservation of Stability

To prove this result, it is convenient to express the behaviour of the environment in generalised Hamiltonian form [33]. The Hamiltonian is formed by defining the generalised momentum as the velocity gradient of the kinetic co-energy and applying a Legendre transformation.

$$\underline{p} \triangleq \partial L / \partial \dot{\underline{q}} \tag{33}$$

$$E_k(\underline{p},\underline{q}) = \underline{p}^t \dot{\underline{q}} - E_k^*(\underline{q},\dot{\underline{q}}) \tag{34}$$

$$H(\underline{p},\underline{q}) = \underline{p}^t \dot{\underline{q}} - L(\underline{q},\dot{\underline{q}}) = E_k + E_p \tag{35}$$

$\underline{p}$: generalised momentum $\quad E_k(\underline{p},\underline{q})$: kinetic energy $\quad H(\underline{p},\underline{q})$: Hamiltonian

The displacement equations are obtained from the momentum gradient of the Hamiltonian.

$$\partial H/\partial \underline{p} = \dot{\underline{q}} \tag{36}$$

Substituting into the Lagrangian form yields the Hamiltonian form of the momentum equations.

$$\dot{\underline{p}} + \partial H/\partial \underline{q} = -\underline{P}(\underline{p},\underline{q}) + \mathbf{J}(\underline{q})^t\underline{F} \tag{37}$$

Rearranging these into the usual causal form:

$$\dot{\underline{q}} = \partial H/\partial \underline{p} \tag{38}$$

$$\dot{\underline{p}} = -\partial H/\partial \underline{q} - \underline{P}(\underline{p},\underline{q}) + \mathbf{J}(\underline{q})^t\underline{F} \tag{39}$$

This formulation has several advantages. The system equations are now in first order form (in contrast to the fundamentally second-order Lagrangian form). The structure of the Hamiltonian form of the equations is preserved under a very broad class of transformations known as canonical transformations [33]. In addition, for this system the Hamiltonian is identical to the total mechanical energy. This latter property can be used to assess system stability, as the total mechanical energy of a stable system may not grow without bound and the total mechanical energy of an assymptotically stable system must decrease. The rate of change of the mechanical energy may be expressed as follows:

$$\underline{H}_q \triangleq \partial H/\partial \underline{q} \tag{40}$$

$$\underline{H}_p \triangleq \partial H/\partial \underline{p} \tag{41}$$

$$dH/dt = \underline{H}_q^t\dot{\underline{q}} + \underline{H}_p^t\dot{\underline{p}} = \underline{H}_q^t\underline{H}_p - \underline{H}_p^t\underline{H}_q - \underline{H}_p^t\underline{P} + \underline{H}_p^t\mathbf{J}^t\underline{F} \tag{42}$$

In the absence of external forces, $\underline{F}$ is zero and this system is isolated. A sufficient condition for stability is then:

$$\underline{H}_p^t\underline{P} > 0 \qquad \text{or} \tag{43}$$

$$\dot{\underline{q}}^t\underline{P} > 0 \tag{44}$$

Now consider a manipulator with the behaviour of the following simple class of impedances:

$$\underline{F} = \underline{K}(\underline{X}-\underline{X}_o) + \underline{B}(\underline{V}) \tag{45}$$

$\underline{K}(\cdot)$: force-displacement relation
$\underline{B}(\cdot)$: force-velocity relation

$\underline{X}_o$ is the vector of desired positions of the manipulator end-effector. In the following it will be assumed to be a constant, corresponding to the maintenance of a fixed posture. If the function relating force to displacement from that posture is restricted so that it has no curl, then a potential energy function can be defined and this simple impedance can be expressed in the following Hamiltonian form:

$$\underline{q} = \underline{X}-\underline{X}_o \tag{46}$$

$$\partial E_p/\partial \underline{q} \triangleq \underline{K}(\underline{q}) \tag{47}$$

$$H(\underline{p},\underline{q}) = E_p(\underline{q}) \tag{48}$$

State equations:

$$\dot{\underline{q}} = \underline{V}(t) \tag{49}$$

$$\dot{\underline{p}} = \partial H/\partial \underline{q} + \underline{B}(\underline{V}(t)) \tag{50}$$

Output equations:

$$\underline{F} = \partial H/\partial \underline{q} + \underline{B}(\underline{V}(t)) \tag{51}$$

The rate of change of the system energy is:

$$dH/dt = \underline{H}_q^t \underline{V} \tag{52}$$

In the absence of imposed motions, $\underline{V}(t)$ is zero and this system is isolated. The rate of change of its total energy is then zero. Although the mechanical energy is non-increasing, no statement can be made about its assymptotic stability. However, one of the assumptions underlying impedance control is that the manipulator is at least capable of stably positioning an arbitrarily small unconstrained mass (i.e. a rigid body) [12]. In Hamiltonian form the equations of motion for a rigid body are:

$$H(\underline{p},\underline{q}) = E_k(\underline{p}) = 1/2\underline{p}^t\mathbf{M}^{-1}\underline{p} \tag{53}$$

M: rigid body inertia tensor

State equations:

$$\dot{\underline{p}} = \underline{F}(t) \tag{54}$$

$$\dot{\underline{q}} = \partial H/\partial \underline{p} \tag{55}$$

Output equations:

$$\underline{V} = \partial H/\partial \underline{p} \tag{56}$$

Note that the rate of change the energy of this system is:

$$dH/dt = \underline{H}_p^t \underline{F} \tag{57}$$

Thus, in common with the simple impedance above, this environmental system has the property that when the force $\underline{F}(t)$ is zero and the system is isolated, its mechanical energy is non-increasing but no statement can be made about its assmyptotic stability.

When the rigid body and the impedance are coupled according to (22) and (23), the equations for the resulting closed system become:

$$H(\underline{p},\underline{q}) = E_k(\underline{p}) + E_p(\underline{q}) \tag{58}$$

$$\dot{\underline{p}} = -\partial H/\partial \underline{q} - \underline{B}(\underline{P}) \tag{59}$$

$$\dot{\underline{q}} = \partial H/\partial \underline{p} \tag{60}$$

The rate of change of the total system energy is:

$$dH/dt = \underline{H}_q^t \underline{H}_p - \underline{H}_p^t \underline{H}_q - \underline{H}_p^t \underline{B} \tag{61}$$

A sufficient condition for stability of the manipulator grasping the rigid body is:

$$\underline{H}_p^t \underline{B} > 0 \qquad \text{or} \tag{62}$$

$$\dot{\underline{q}}^t \underline{B} > 0 \tag{63}$$

Now consider the stability of the system formed when the simple impedance described by (49), (50) and (51) is coupled to the more general environment described by (29), (38) and (39) through the coupling equations (22) and (23). In the following, subscript 1 refers to the manipulator and subscript 2 refers to the environment. The total system energy is:

$$H_{total} = H_1 + H_2 \tag{64}$$

its rate of change is:

$$dH_{total}/dt = \underline{H}_{1p}^t \dot{\underline{p}}_1 + \underline{H}_{1q}^t \dot{\underline{q}}_1 + \underline{H}_{2p}^t \dot{\underline{p}}_2 + \underline{H}_{2q}^t \dot{\underline{q}}_2 \tag{65}$$

$$= \underline{H}_{1q}^t \mathbf{J} \underline{H}_{2p} - \underline{H}_{2p}^t \underline{H}_{2q} - \underline{H}_{2p}^t \underline{P} - \underline{H}_{2p}^t \mathbf{J}^t \underline{H}_{1q} - \underline{H}_{2p}^t \mathbf{J}^t \underline{B} + \underline{H}_{2q}^t \underline{H}_{2p} \tag{66}$$

Eliminating terms:

$$dH_{total}/dt = -\underline{H}_{2p}^t \underline{P} - \underline{H}_{2p}^t \mathbf{J}^t \underline{B} \tag{67}$$

Using (29) and (38) the last term in (67) can be written in terms of the velocity at the interaction port.

$$dH_{total}/dt = -\dot{\underline{q}}_2^t \underline{P} - \dot{\underline{q}}_1^t \underline{B} \tag{68}$$

Thus the sufficient conditions (44) and (63) for stability of each of the two individual systems are also sufficient to guarantee stability of the coupled system. Intuitively, this makes physical sense because the non-energic coupling does not generate energy, thus there is no mechanism through which the total mechanical energy could grow without bound, and the frictional elements, however small, ensure that the total mechanical energy always decreases.

Summarising briefly, this discussion has shown that structuring the dynamic behaviour of a manipulator to be causally compatible with its environment has desirable stability properties. Note that the proof can be extended to more general forms of the target impedance without losing the fundamental result.

7. Application to a Biological System

Because of the generality of the physical equivalence conjecture these concepts can be applied to complex biological systems. How well do they describe observed behaviour? If the skeleton is modelled as a collection of kinematically constrained rigid bodies, then it is properly described as an admittance. Consequently, by the reasoning above, the neuromuscular system should behave as an impedance [11].

Despite the complexity of the thermodynamically non-conservative physiological processes underlying muscle contraction, under normal physiological conditions the external behaviour of a single muscle exhibits a relation between force and displacement similar to that of a spring [8, 26]. A growing body of literature in the neurosciences [1, 2, 4, 6, 15, 21, 25, 28, 31, 32] has investigated the influence of this "spring-like" behaviour on the control of movement. Indeed, one prominent and (to date) successful hypothesis [5, 10,22] explains one of the principal functions of the spinal reflex arcs (involving muscle spindles and Golgi tendon organs) as preserving the spring-like behaviour of an individual muscle in the face of perturbing effects. Furthermore, the relation between force and velocity produces a behaviour similar to that of a frictional element [3, 9, 19, 20], and thus a single muscle does, in fact, exhibit the behaviour of an impedance of the form of (45).

When muscles act in coordinated synergy, there is no guarantee that the behaviour of the complete neuromuscular system will be equally simple. For example, the presence of intermuscular spinal reflex arcs could introduce a relation between force and displacement with non-zero curl, or a relation between force and velocity with non-zero curl [11]. Such a system would still be an impedance, but would not enjoy the stability properties discussed above. However, recent experiments by the author and colleagues [21] have investigated the patterns of postural stiffness of the human upper extremity. Under steady state postural conditons, the anti-symmetric component of the stiffness was negligible in comparison to the symmetric component of the stiffness (see fig. 1) verifying that under these conditions the entire neuromuscular system

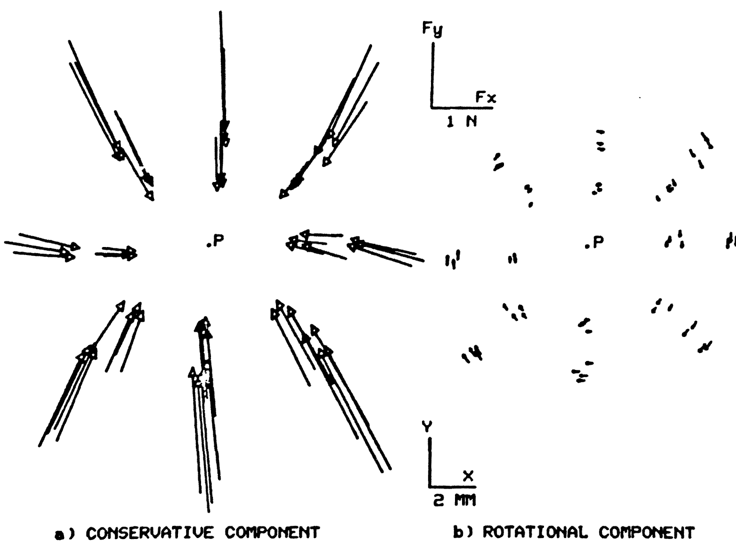

Figure 1: While human subjects maintained a fixed posture of the upper extremity, a series of small (approximately 4 to 8 mm in magnitude) displacements were imposed on the hand and the steady state postural restoring force generated by the neuromuscular system in response was measured. The postural stiffness matrix was estimated by multivariable regression of between 50 and 60 observations of the force vectors onto the corresponding displacement vectors. This figure shows graphical representations of the symmetric (conservative or spring-like) component and the antisymmetric (rotational or curl) component of the postural stiffness. In these diagrams the two components are represented by drawing the force vectors obtained by multiplying each of the imposed displacement vectors by the symmetric (part a) and antisymmetric (part b) components of the postural stiffness. Each force vector is drawn with its tip at the tip of the corresponding displacement vector. For clarity, the displacement vectors are not shown. The nominal hand posture is at point P in each diagram.

of the upper extremity behaves as a simple impedance of the form of (45). Note that this behaviour requires that either the intermuscular reflex feedback is non-existent or that it is exquisitely balanced [11]. For example, the gain of the reflex pathways relating torque about the elbow to rotation of the shoulder must be identical to that relating torque about the shoulder to rotation of the elbow. These results suggest that despite the evident complexity of the neuromuscular system, coordinative structures in the central nervous system go to some lengths to preserve the simple "spring-like" behaviour of the single muscle at the level of the complete neuromuscular system.

If such finely tuned coordinative structures exist, what is their purpose? The analysis presented in this paper offers one explanation of the benefits of imposing the behaviour of a generalised spring on the neuromuscular system. If the curl of the force displacement relation is zero then the stability of the isolated limb is guaranteed with even the most modest frictional effect. Furthermore, when the limb grasps an external object -- even an object as complicated as another limb on another human -- then if that object is stable in the sense described above, the stability of the coupled system is again guaranteed.

8. Conclusion

The approach outlined in this paper offers a new perspective on the control of complex systems such as the primate upper extremity. An unique feature of the approach is that it is firmly based in physical systems theory. One important aspect of the dynamic equations of a physical system is their structure. If a manipulator is to be physically compatible with its dynamic environment then its behaviour should complement that of the environment. In the most common case in which the environment has the behaviour of a generalised mechanical admittance, the manipulator must have the behaviour of a generalised mechanical impedance, and its controller should not attempt to impose any other behaviour.

Imposing appropriate structure on the dynamic behaviour of a manipulator can result in superior stability properties. It must again be stressed that in general the stability of a dynamic system is jeopardised when it is coupled to a stable dynamic environment. In contrast, in this paper it was shown that if the force displacement behaviour of a manipulator has the structure of a generalised spring then the stability of the manipulator is preserved when it is coupled to a stable environment. Experiments to date indicate that the behaviour of the neuromuscular system of the human upper extremity has precisely this structure.

Control of a complex system is not exclusively a matter of preserving stability; acceptable performance must also be achieved. A clear definition of "acceptable performance" may prove to be elusive, but one desirable feature is that the manipulator should have a sufficiently rich repertoire of behaviour. In that context it is interesting to note that the impedance control strategies discussed in this paper give the manipulator the behaviour of a set of coupled nonlinear oscillators. Coupled nonlinear oscillators exhibit a prodigious richness of behaviour, and recent research has shown that some of their behavioural peculiarities are qualitatively similar to aspects of coordinated human movement [29, 30].

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